

Handelskammaren
I sydsvenska företags intresse

PQ Riksfinal 2018

Part 1

Time: 60 min – 6 Questions

Max points: 18 (3p/question).

Allowed tools: Paper, pencil and rubber (no calculator).

Write your solutions on separate pieces of paper. One sheet per question. Write your team name on each sheet of paper.

Please show full working for your answers to Part 1.

1. Road trip

On Monday, Mukesh travelled x km at a constant speed of 90 km/h. On Tuesday, he travelled on the same route at a constant speed of 120 km/h. His trip on Tuesday took 16 minutes less than his trip on Monday. What is x ?

2. Lost in the forest

You are lost in the rainforest and have eaten a deadly poisonous mushroom by mistake. In order to save your life, you need to lick the antidote which is secreted (made) from a certain type of frog. Unfortunately it is only the female who produces the antidote. In this rainforest there are as many male frogs as there are female frogs and they look identical, but the male frog has a distinct croak (sound). You only have seconds left to live when you suddenly see one of these frogs to your right sitting on a tree stump. Then, to your left you hear the special call of the male frog and, just after the croak stops, turn to see two identical frogs close together in a cleaning.

Which way should you turn in order to have highest chance of licking a female frog and be saved? (You can lick two frogs at the same time)

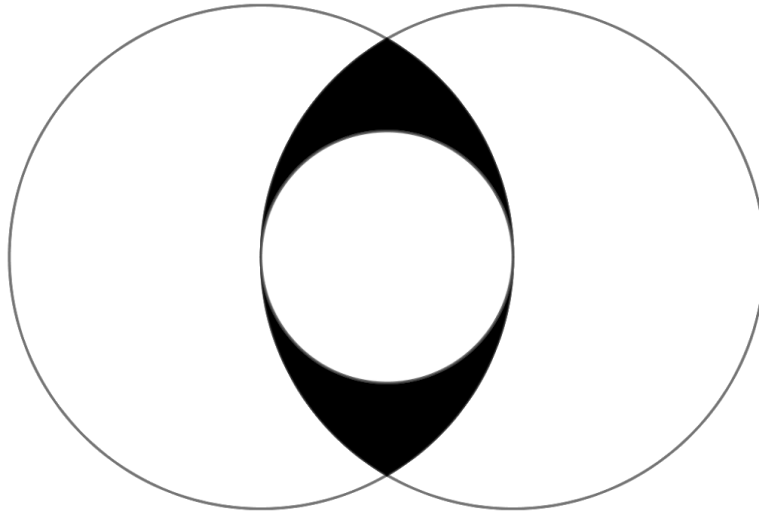
3. More power to the PQ

A number p is chosen at random from $\{11, 13, 15, 17, 19\}$ and a number q is chosen at random from $\{1999, 2000, 2001, 2002, \dots, 2018\}$.

What is the probability that p^q ends with a 1?

4. In a squeeze

A small circle is inscribed between two larger circles as in the figure below. If the radius of the larger circles is a and the diameter of the smaller circle is also a , then what is the area of the shaded area? Give the exact answer and simplify as far as possible.



5. The Pirates

Once upon a time, five pirates were caught and chained to different places on an island. On the island were 6 identical trees. One of the pirates had some treasure with them which they decided to bury at once. Then, as if by chance, a large wave washed over the island carrying with it all the pirates, but leaving the trees and the buried treasure. Later, they decide to return to find their treasure. They fly over in a plane looking for the island from above. Unfortunately they see lots of different islands all of the same shape and all with 6 trees, but arranged in different patterns. Each pirate claims to have seen a different number of trees from 2 to 6. The pirate who saw two trees, was the one who buried the treasure.

What could the pattern of trees on the island look like and where is the treasure buried?

6. The Mascot

The Mascot

One day, all the pupils at Borgar decide to line up in one long line and march to the rival school, Petri, at constant speed. Whilst the pupils are marching, Borgar's mascot, The Borgar Bear, runs from the President of the Students' Union at the back of the line, to the Flag Bearer, who is at the front of the line and then back to the President who is now where the Flag Bearer was from the start.

How far has the mascot run in total if the line is 100 m long? Give the exact answer and simplify as far as possible.

Part 2.

Time : 30 min – 6 questions Max: 12 points (2p/question).

Allowed tools: Only paper, pencil and rubber

Write your team name on all sheets of paper.

Answers only are required for Part 2.

1. Mera powers

If $p = 2^{129} \cdot 3^{81} \cdot 5^{128}$, $y = 2^{126} \cdot 3^{81} \cdot 5^{129}$, $t = 2^{126} \cdot 3^{82} \cdot 5^{128}$ and $h = 2^{125} \cdot 3^{82} \cdot 5^{129}$, arrange the numbers p , y , t & h from smallest to largest.

2. Fractious Coffee

Carina pours 60 ml coffee into an 120 ml cup and 60 ml of cream into a second cup of the same size. She then pours half the coffee from the first cup into the second and, after stirring thoroughly, pours half the liquid in the second cup back into the first.

What fraction of the liquid in the first cup is now cream?

3. Housing Bubble

Ulf bought a house for a million crowns. During the following eight years, the value of the house increased, compared with the preceding year, by a ninth, an eighth, a seventh, a sixth, a fifth, a fourth, a third and finally by a half.

What is the value of Ulf's house now?

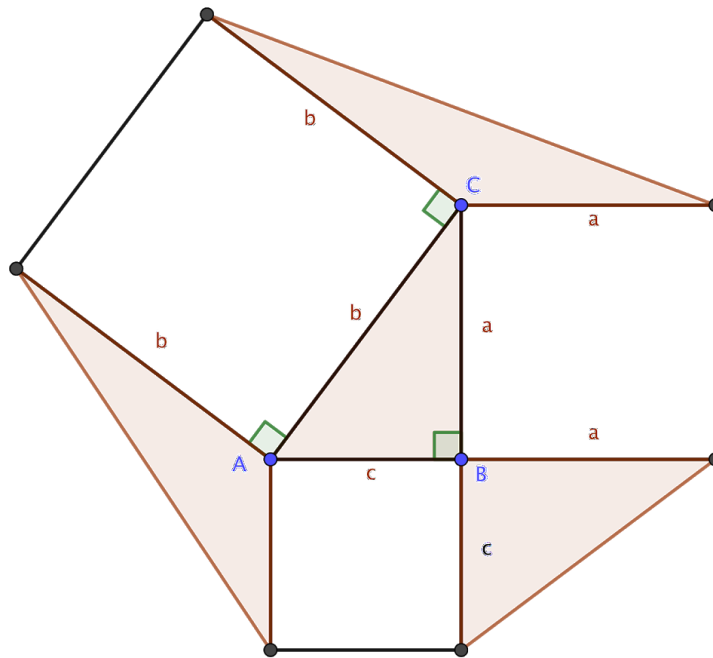
4. More power to the 2

Aila chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. Freyja chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$. Marie chooses an integer at random from the set $\{2, 4, 6, 8, 10\}$.

What is the probability that the product of their three integers is not a power of 2?

5. Pythagoras Puzzle

In the diagram below, triangle ABC is a right-angled triangle with $\angle C = 90^\circ$. If $a = 4$, $b = 5$ och $c = 3$, calculate the fraction of the figure which is shaded. Simplify as far as possible.



6. We are the Heroes...

In a dystopian future, the world has been taken over by the evil Petrius. Our heroes Arvid, Linn and Theodor are left. Unfortunately for us, Petrius has managed to imprison them too. However during a random act of kindness, Petrius gives one of them a chance to escape if they can solve a riddle. They choose Linn since she is the best one at solving logic problems. Arvid and Theodor attach a microphone to Linn so they can follow the conversation. Linn is released and runs away through a numbered corridor. She is then faced with a locked door with a number pad. This is the conversation Arvid and Theodor hear.

Petrius: The code consists of three positive integers P Y Q whose product is equal to 36. Q is greater than or equal to Y and Y is greater than or equal to P . The sum of the integers is the same as the number above the corridor you just came through.

Linn: I know the number above the corridor, but I still don't know the code. Please Petrius, just give me one more clue.

Petrius: Okay then. Q is not equal to either P nor Y .

Linn: Now I know the code!

Linn types in the code and rushes to freedom.

What is the code P Y Q ?

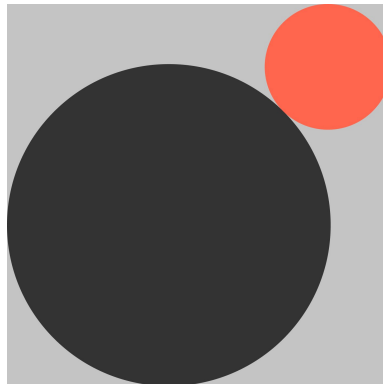
Part 3

Part 3 Max time: 10 min

Decider Question

Calculator and other digital tools, not allowed.

If two teams have the same points after Parts 1 and 2, then this counts as the decider question. In the first place, correct answers are considered. Secondly the team that solved it in the shortest possible time will be placed before the other team(s).



Source: brilliant.org

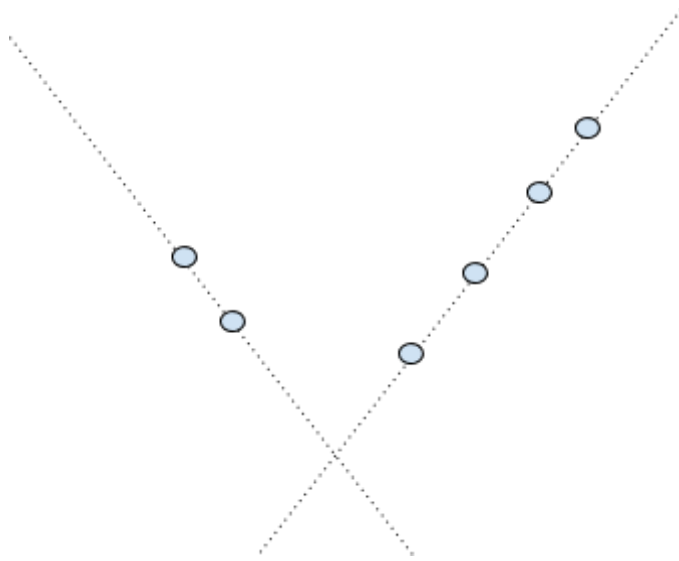
Two circles are inscribed inside a square as shown. Let the radius of the larger circle be $2r$ and the radius of the smaller circle be r .

Find the area of the square in terms of r .

Key

Part 1

1. Till vänster ($P = \frac{2}{3}$ istället för $\frac{1}{2}$).
2. $x = 96\text{km}$
3. $\frac{2}{5}$ eller 40%
4. Andelen = $\frac{5\pi}{12}a^2 - \frac{\sqrt{3}}{2}a^2$
5. $100 + \frac{200}{\sqrt{2}}$ m
6. T.ex.



Part 2

1. t, y, h, p
2. 5 miljoner
3. $\frac{2}{5}$
4. 98/125
5. 12/37
6. 2 2 9

Part 3

Answer = $\frac{9(1+\sqrt{2})^2 r^2}{2}$ eller motsvarande